International Journal of Computer Mathematics
Vol. 00, No. 00, Month 200x, 1–3

Solving the Black-Schole's equation in an easy way

A. B. Cde†, F. Ghi*‡ and J. Klm§

†Department of Mathematics, ABC State University
ABC, NY 67890-1010, USA

‡Departament de Finances, BCDE Université
XYZ, 77777 FGH Cedex, France

§LMN Internacional Corporación Banco, E-10234, FGH, Spain

(*Received 00 Month 200x; revised 00 Month 200x; in final form 00 Month 200x*)

This paper studies a completely new method...

Keywords: Black-Schole equation; Finite difference; Adaptation; Finance derivatives; Option market; Hedging strategies

AMS Subject Classification: 65K20; 65M50; 35K65; 35B40

Dedicated to A. Bcd, E. Fgh, I. Jkl, M. Nop, and Q. Rst for their contributions to the mathematical finance

1. Introduction

Many mathematical models established for important physical processes have the property that they may develop singularities in a finite time T. A typical example is the nonlinear reaction-diffusion equations of quenching type, in which the nonlinear forcing term blows up, or the solution quenches and extinguishes, in a finite time when certain environmental parameters exceed their limits.

$$u_{xx}(x_i, y_j, t) \approx k_b u(x_i + k_f, y_j, t) k_b k_f (k_b + k_f)/2, \quad (1.1)$$

$$u_{yy}(x_i, y_j, t) = h_b u(x_i, y_j + h_f, t) h_b h_f (h_b + h_f)/2. \quad (1.2)$$

Let $w(t)$ be a numerical solution to (1.1), (1.2) through a semidiscretization in space.
In principle [1], a finance model is $ABC = \delta$.

Corresponding author. Email: ABC@defgh.edu

International Journal of Computer Mathematics
ISSN: 0020-7160 print/ISSN 1029-0265 online © 200x Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/0020716YYxxxxxxxx

2. Initial achievement

By denoting $x/a = \tilde{x},\ y/b = \tilde{y}$, equation (1.1) can be formulated to $X = Y$.

$$\frac{\partial^2 S}{\partial \theta^2} = 123.567. \tag{2.3}$$

3. Further improvements

The positivity and monotone increasing properties have been characterized as the most important features of the solution of the equation (1.1)-(2.3) as stated in [1, 2, 4]. They reflect proper natural behaviors of the physical processes modeled by the differential equations. Thus, it is important to preserve these basic properties of the numerical solution. For the sack of simplicity, we assume that $\beta \geq \alpha > 0$. For the splitting scheme (2.3), we have

LEMMA 3.1 *Let* $h = \min_{b,f}\{h_b, h_k, k_b, k_f\}$ *for all internal points* (x_i, y_j) *at the time* t*. If* $A > 0$ *then* $I + \tau_k C$ *is nonsingular.*

Proof The conclusions are straightforward due to properties of P and R. □

Another obvious fact is the following. Combining above results we obtain the following theorem.

THEOREM 3.2 *For any beginning step* ℓ *if*
(i) $a = b$ *for all* $k \geq \ell$,
(ii) $c = d$ *for all* $\tau_k > 0$,
then the sequence $\{v_k\}_{k \geq \ell}$ *produced by the splitting scheme* (2.3) *is convergent.*

Proof The conclusions are straightforward due to properties of Q and S. □

4. Numerical experiments

The initial value u_0 must be set to zero. This is discussed in [3] by Rogers in 2007. We may further introduce a new method.

Let $D = (0, a) \times (0, b)$. Consider the following singular nonlinear reaction-diffusion problem

$$\begin{aligned}
&\left(x^2 + y^2 + \epsilon\right) u_t = 0, \quad (x, y) \in D,\ 0 < t < T,\\
&u(x, y, 0) = u_0, \quad (x, y) \in D;\\
&u(0, y, t) = u(a, y, t) = u(x, 0, t) = u(x, b, t) = 0, \quad 0 < t < T,
\end{aligned}$$

where $\epsilon = 10^{-3}$.

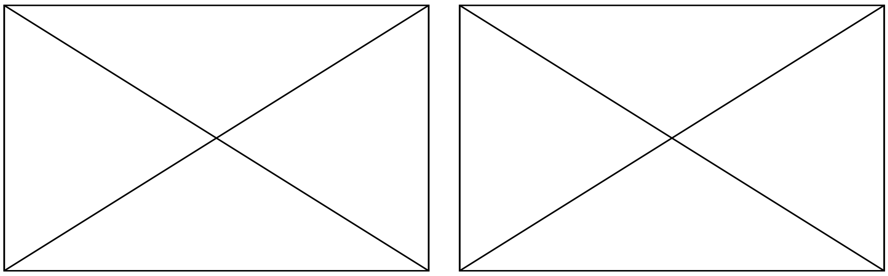

Figure 1. LEFT: A projection. RIGHT: A contour map.

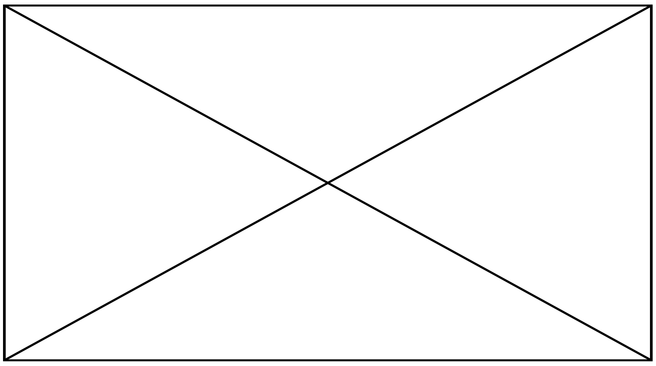

Figure 2. A 3-dimensional view of the derivative.

Table 1. Values of α used in the experiments.

$a \setminus g_k$	0.01	1	4	0.618
1	1.4122	1.29065	1.17999	10
0.5	1.4132	1.33757	1	
0.8	1.6132	1.3375709	9.9	0.1

Acknowledgements

The research of the first author was supported in part by a REP research grant under No. 032-1318-06. The authors sincerely thank the funding agencies for the supports.

References

[1] Arnold, W., Smith, W. and Zell, R. D., 2003, Approaches of general moving meshes. *Appl. Fin. Math.*, **147**, 1021-1038.
[2] Golub, G. H. and Van Loan, C. F., 1996, *Matrix Computations,* Johns Hopkins University Press, Baltimore and London, 3rd Ed.
[3] Rogers, K. Jr., 2007, A stochastic analysis of some frequently used financial models, preprint.
[4] Sheng, Q. and Khaliq, A. Q. M., 2001, Linearly implicit adaptive schemes for singular reaction-diffusion equations, Chapter 9. *Adaptive Method of Lines,* (edi. A. Vande Wouwer, Ph. Saucez and W. E. Schiesser) Capman & Hall/CRC, London and New York.