

Math 3326  
Spring Semester 2009  
Problem Set #1

1. Show that if  $p$  is a continuously differentiable function of one variable, the first-order partial differential equation

$$u_t = p(u)u_x$$

has a solution implicitly defined by

$$u(x, t) = \varphi(x + p(u)t),$$

in which  $\varphi$  can be any continuously differentiable function of one variable. Use this idea to determine (perhaps implicitly) a solution of each of the following equations:

- (a)  $u_t = ku_x$  (with  $k$  being a nonzero constant)
  - (b)  $u_t = uu_x$
  - (c)  $u_t = \cos(u)u_x$
  - (d)  $u_t = u \sin(u)u_x$ .
2. Show that  $u(x, y) = \ln((x - x_0)^2 + (y - y_0)^2)$  satisfies  $u_{xx} + u_{yy} = 0$  for all pairs  $(x, y)$  of real numbers except  $(x_0, y_0)$ .
3. Let  $k$  be a positive constant. Let

$$u(x, t) = \frac{1}{2\sqrt{\pi kt}} \int_{-\infty}^{\infty} e^{-(x-\xi)^2/4kt} f(\xi) d\xi,$$

in which  $f$  is continuous on the real line. Show that  $u_t = ku_{xx}$  for  $-\infty < x < \infty$ ,  $t > 0$ . Also determine  $u(x, t)$  when  $f(x) = 1$  for all real  $x$ . Hint: Use a change of variable and the standard result that

$$\int_{-\infty}^{\infty} e^{-w^2} dw = \sqrt{\pi}.$$