Math 3326 Quiz #9

SPRING SEMESTER 2009

Name SOLUTIONS

1. Solve the heat equation

$$u_t = u_{xx}$$
$$u_x(0) = u_x(\pi) = 0$$
$$u(x,0) = x \quad (0 < x < \pi).$$

From class, the solution to $u_t = k u_{xx}$
$u_x(0,t) = u_x(L,t) = 0$
$u(x,0) = f(x)$
is $u(x,t) = \sum_{n=0}^{\infty} c_n \cos\frac{n\pi x}{L} e^{-\frac{kn^2\pi^2}{L^2}t}$ where $c_0 = \frac{1}{L}\int_0^L f(x)\,dx$

& $c_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx \quad (n=1,2,\ldots)$

In this case, $L = \pi$ & $f(x) = x$ so

$$c_0 = \frac{1}{\pi}\int_0^\pi x\,dx = \frac{1}{\pi}\cdot\frac{\pi^2}{2} = \frac{\pi}{2}.$$

& $c_n = \frac{2}{\pi}\int_0^\pi x\cos nx\,dx$ $\quad u = x \quad dv = \cos nx\,dx$
$\quad du = dx \quad v = \frac{\sin nx}{n}$

$$= \frac{2}{\pi}\left[\frac{x\sin nx}{n}\Big|_0^\pi - \frac{1}{n}\int_0^\pi \sin nx\,dx\right] = -\frac{2}{\pi n}\left[-\frac{\cos nx}{n}\right]_0^\pi$$

$$= \frac{2}{\pi n^2}\left[(-1)^n - 1\right] \text{ so } c_{2n} = 0 \text{ & } c_{2n-1} = \frac{-4}{\pi(2n-1)^2}.$$

$$\therefore \boxed{u(x,t) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}e^{-(2n-1)^2 t}}$$