

# Math 3326 Quiz #8

SPRING SEMESTER 2009

Name SOLUTIONS

1. Find the Fourier cosine series for

$$f(x) = \begin{cases} 0 & \text{if } 0 \leq x < \frac{\pi}{2} \\ 1 & \text{if } \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

on  $[0, \pi]$ . Simplify your coefficients as much as possible.

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx = \frac{2}{\pi} \int_{\pi/2}^{\pi} dx = \frac{2}{\pi} (\pi - \frac{\pi}{2}) = 1$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{\pi/2}^{\pi} \cos nx dx = \frac{2}{\pi} \left[ \frac{\sin nx}{n} \Big|_{\pi/2}^{\pi} \right]$$

$$= \frac{-2}{\pi n} \sin n \frac{\pi}{2} \quad \text{so } a_{2n} = 0 \quad \& \quad a_{2n-1} = \frac{-2}{\pi(2n-1)} \sin \frac{(2n-1)\pi}{2}$$

$$= \frac{-2}{\pi(2n-1)} (-1)^{n+1}$$

$$= \frac{2(-1)^n}{\pi(2n-1)}$$

$$\therefore \text{FCS } f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n \cos(2n-1)x}{2n-1}$$