Math 3326 Quiz #7

SPRING SEMESTER 2009

Name SOLUTIONS.

1. Find the Fourier series of $f(x) = \begin{cases} 1 & -\pi \le x < 0 \\ 2 & 0 \le x \le \pi. \end{cases}$ .Simplify your coefficients as much as possible.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} dx + \int_{0}^{\pi} 2dx\right] = \frac{1}{\pi}\left[\pi + 2\pi\right] = 3$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0}\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{\sin nx}{n}\Big|_{-\pi}^{0} + \frac{2\sin nx}{n}\Big|_{0}^{\pi}\right] = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0}\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2\sin nx\,dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos nx}{n}\Big|_{-\pi}^{0} - \frac{2\cos nx}{n}\Big|_{0}^{\pi}\right] = \frac{1}{\pi}\left[-\frac{1}{n} + \frac{(-1)^n}{n} - \frac{2(-1)^n}{n} + \frac{2}{n}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n} - \frac{(-1)^n}{n}\right] \text{ so } b_{2n} = 0 \text{ and } b_{2n-1} = \frac{2}{\pi(2n-1)}$$

$$\therefore \ FSf(x) = \frac{3}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1} \quad (*)$$

2. From 1, evaluate the series $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{2n-1}$.

By our convergence theorem, $FSf(x)\Big|_{x=\frac{\pi}{2}} = \dfrac{f(\frac{\pi}{2}^+) + f(\frac{\pi}{2}^-)}{2} = 2.$

Let $x = \pi/2$ in the term in the right-hand side of $(*)$:

$$\frac{3}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)\frac{\pi}{2}}{2n-1} = \frac{3}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$$

$$\therefore \quad 2 = \frac{3}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1} \Rightarrow \frac{1}{2} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$$