Math 3326 Quiz #6

SPRING SEMESTER 2009

Name SOLUTIONS

1. Consider the PDE $u_{xx} + 2u_{xy} + 5u_{yy} + 3u = 0$. Using $\eta(x, y) = x + y$ in both transformations, find the canonical form of this elliptic equation.

Here, $A=1, B=1, C=5$ so $B^2-AC = 1-5 = -4 < 0$ so this equation is elliptic on $\mathbb{R}^2$.

Now $\frac{dy}{dx} = \frac{B\eta_x + C\eta_y}{A\eta_x + B\eta_y} = \frac{B+C}{A+B} = \frac{6}{2} = 3$ so $y = 3x + k$

Take $\xi(x,y) = y - 3x$

Then $a = A\xi_x^2 + 2B\xi_x\xi_y + C\xi_y^2 = 1(9) + 2(-3)(1) + 5(1) = 9 - 6 + 5 = 8$

$b = 0$ (as necessary)

$c = A\eta_x^2 + 2B\eta_x\eta_y + C\eta_y^2 = 1 + 2 + 5 = 8$

So the transformed equation is

$$8w_{\xi\xi} + 8w_{\eta\eta} + 3w = 0$$

or $$\boxed{w_{\xi\xi} + w_{\eta\eta} + \frac{3}{8}w = 0}$$