Math 3326 Quiz #6

FALL SEMESTER 2008

Name SOLUTIONS

1. Find the Fourier cosine series for $f(x) = \sin x$ on $[0, \pi]$. Simplify your coefficients as much as possible.

$$a_0 = \frac{2}{\pi}\int_0^\pi \sin x\,dx = -\frac{2}{\pi}\cos x\Big|_0^\pi = \frac{4}{\pi}$$

$$a_1 = \frac{2}{\pi}\int_0^\pi \sin x\cos x\,dx = \frac{1}{\pi}\int_0^\pi \sin 2x\,dx = 0$$

For $n>1$, $a_n = \frac{2}{\pi}\int_0^\pi \sin x\cos nx\,dx = \frac{1}{\pi}\int_0^\pi [\sin(n+1)x - \sin(n-1)x]dx$

$$= \frac{1}{\pi}\left[\frac{-1}{n+1}\cos(n+1)x + \frac{1}{n-1}\cos(n-1)x\right]_0^\pi = \frac{1}{\pi}\left[\frac{(-1)^n}{n+1} + \frac{(-1)^{n-1}}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right]$$

$$= \frac{1}{\pi}\left[\frac{-2(-1)^n}{n^2-1} - \frac{2}{n^2-1}\right] = \frac{-2}{\pi(n^2-1)}[(-1)^n + 1]$$

Hence $a_{2n-1} = 0$ and $a_{2n} = \frac{-4}{\pi(4n^2-1)}$

Hence

(*) FCS $f(x) = \sin x$ on $[0,\pi]$ $= \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nx}{4n^2-1}$

2. From Problem 1, evaluate the infinite series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n^2-1}$.

In (*), let $x = \pi/2$

Then $1 = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\pi}{4n^2-1} = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$.

Hence $-\frac{\pi}{4}\left(1 - \frac{2}{\pi}\right) = \sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$

ie/ $\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1} = \frac{2-\pi}{4}$.