Math 3326 Quiz #5

SPRING SEMESTER 2009

Name SOLUTIONS

1. Find the solution of the Cauchy wave equation

$$u_{tt} = 4u_{xx}$$
$$u(x,0) = x^2, u_t(x,0) = \sin(2x).$$

Simplify your answer as much as possible.

d'Alembert's formula is

$$u(x,y) = \frac{1}{2}[f(x+\alpha t)+f(x-\alpha t)] + \frac{1}{2\alpha}\int_{x-\alpha t}^{x+\alpha t} g(u)\,du$$

Here, $f(x)=x^2$ and $g(x)=\sin(2x)$, $\alpha=2$, so that

$$u(x,y) = \frac{1}{2}[(x+2t)^2+(x-2t)^2] + \frac{1}{4}\int_{x-2t}^{x+2t} \sin 2u\,du$$

$$= \frac{1}{2}[x^2+4xt+4t^2+x^2-4xt+4t^2] - \frac{1}{8}\cos 2u\Big|_{x-2t}^{x+2t}$$

$$= x^2+4t^2-\frac{1}{8}\cos(2x+4t)+\frac{1}{8}\cos(2x-4t)$$

$$= x^2+4t^2+\frac{1}{4}\sin 2x \sin 4t$$