

Math 3326 Quiz #4

SPRING SEMESTER 2009

Name SOLUTIONS

1. Consider the PDE

$$u_{xx} - 2u_{xy} - 3u_{yy} = 0. \quad (1)$$

(a) Verify that this PDE is hyperbolic in the region $D = \mathbb{R}^2$.

$$A=1, B=-1, C=-3 \text{ so } B^2-AC = 1+3=4 > 0$$

$$\frac{B \pm \sqrt{B^2-AC}}{A} = \frac{-1 \pm 2}{1}$$

(b) Solve the two characteristic equations associated with this PDE for the change of variables $\xi = \xi(x, y)$ and $\eta = \eta(x, y)$.

$$\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = -3$$

$$\Rightarrow y = x + c_1$$

$$\Rightarrow y = -3x + c_2$$

$$\text{so let } \xi(x, y) = y - x$$

$$\text{so let } \eta(x, y) = y + 3x$$

(c) Using the change of variables, solve equation (1) in terms of two arbitrary, twice differentiable functions. Show all of your work (use the back page of this page as well).

The transformed equation simplifies to

$$w_{\xi\eta} = 0$$

so $w_{\xi} = f(\xi) \Rightarrow w(\xi, \eta) = F(\xi) + G(\eta)$, where F & G are arbitrary, twice continuously differentiable functions.

Hence

$$u(x, y) = F(y-x) + G(y+3x)$$

Solves the above PDE.