

Math 3326 Quiz #3

SPRING SEMESTER 2009

Name SOLUTIONS

1. Classify each of the following second-order PDE's according to whether they are hyperbolic, parabolic, or elliptic in the plane \mathbb{R}^2 :

(a) $2u_{xx} - 4u_{xy} + 7u_{yy} - u = 0$

Here, $A=2, B=-2, C=7$ so $B^2-AC = 4 - (2)(7) = -10 < 0$
so this equation is elliptic everywhere in \mathbb{R}^2 .

(b) $u_{xx} - 2\cos(x)u_{xy} - \sin^2(x)u_{yy} = 0$.

Here, $A=1, B=-\cos x, C=-\sin^2 x$ so $B^2-AC = \cos^2 x + \sin^2 x = 1$
so this equation is hyperbolic everywhere in \mathbb{R}^2 .

- (c) $yu_{xx} + 2(x-1)u_{xy} - (y+2)u_{yy} = 0$. For this particular PDE, identify and graph the curve in the xy -plane that defines the region D_1 for which this equation is parabolic. Then, on your graph, identify the region D_2 where this equation is hyperbolic and the region D_3 where this equation is elliptic.

Here, $A=y, B=x-1, C=-(y+2)$ so

$$B^2-AC = (x-1)^2 + y(y+2) = (x-1)^2 + (y^2+2y) \\ = (x-1)^2 + (y+1)^2 - 1$$

When $B^2-AC=0$ (the parabolic case), we see that
 $(x-1)^2 + (y+1)^2 = 1$

This is a circle of radius 1 centered at $(1, -1)$.

