Math 3326 Quiz #2

SPRING SEMESTER 2009

Name SOLUTIONS

1. Find the general solution to the first-order PDE

$$u_x + yu_y - u = 0.$$

In your solution, identify and solve the characteristic equation and the transformed equation in order to solve the above PDE.

CE: $\frac{dy}{dx} = y \Rightarrow \frac{dy}{y} = dx \Rightarrow \ln y = x + k$

$\Rightarrow y = e^{x+k} = Ce^x$

so take $\eta(x,y) = ye^{-x}$ & $\xi(x,y) = x$.

ie/ the characteristic curves are the exponentials $y = Ce^x$ where C is an arbitrary constant.

TE: $w_\xi - w = 0$

Multiply by $e^{-\int d\xi} = e^{-\xi}$: $(e^{-\xi}w)_\xi = 0$

Integrate: $e^{-\xi}w = f(\eta)$ so $w = e^{\xi}f(\eta)$

$\therefore$ the general solution to $u_x + yu_y - u = 0$ is

$$u(x,y) = e^x f(ye^{-x}).$$

2. Now solve, if possible, the above first-order equation with Cauchy data $u(x,y) = 2$ on $y = 3e^x$.

We want: $2 = u(x, 3e^x) = e^x f(3e^x \cdot e^{-x}) = e^x f(3)$

This equation is impossible so this Cauchy problem has no solutions.

3. Now solve, if possible, the same first-order equation but this time with Cauchy data $u(x,y) = e^x$ on the curve $y = e^x$.

Now, we want: $e^x = u(x, e^x) = e^x f(e^x \cdot e^{-x}) = e^x f(1)$ so $f(1) = 1$.

In this case, there are infinitely many solutions to this Cauchy problem; specifically, $u(x,y) = e^x f(ye^{-x})$, where f is an arbitrary differentiable function satisfying $f(1)=1$

ie/ $f(t) = t$ $f(t) = t^2$ $f(t) = \cos(t-1)$ etc.