Math 3326 Quiz #2

FALL SEMESTER 2008

Name SOLUTIONS

1. Consider the first-order linear partial differential equation

$$4u_x + 7u_y - u = 2.$$

(a) Find the general solution to this equation. $\xi = x;\ \eta = \eta(x,y)$

Transformed Equation: $4w_\xi - w = 2$ or $w_\xi - \frac{1}{4}w = \frac{1}{2}$

Multiply by $e^{-\frac{1}{4}\xi}$: $(e^{-\frac{1}{4}\xi}w)_\xi = \frac{1}{2}e^{-\frac{1}{4}\xi}$

and solve to get $e^{-\frac{1}{4}\xi}w = -2e^{-\frac{1}{4}\xi} + g(\eta)$

ie/ $w(\xi,\eta) = -2 + e^{\frac{1}{4}\xi}g(\eta)$

Characteristic Equation: $\frac{dy}{dx} = \frac{7}{4}$ so $y = \frac{7}{4}x + C$

or $4y - 7x = K$; take $\eta(x,y) = 4y - 7x$ (so $4y - 7x = K$ are the characteristic curves)

$\therefore$ the solution to $4u_x + 7u_y - u = 2$ is

$$\boxed{u(x,y) = -2 + e^{\frac{1}{4}x}g(4y-7x)}$$

(b) Is there a solution of this problem with the Cauchy data

$$u(x,y) = \sin x \text{ on the line } y = 5x.$$

If so, find this solution explicitly. If there is no solution, explain why not.

We want: $\sin x = u(x,5x) = -2 + e^{\frac{1}{4}x}g(4(5x) - 7x)$

$= -2 + e^{\frac{1}{4}x}g(13x)$

so $g(13x) = e^{-\frac{1}{4}x}(\sin x + 2)$

$(x \to \frac{x}{13})$ $\Rightarrow g(x) = e^{-\frac{x}{52}}(\sin\frac{x}{13} + 2)$

$\therefore$ the solution to this Cauchy problem is given by

$$u(x,y) = -2 + e^{\frac{1}{4}x}g(4y-7x) = -2 + e^{\frac{1}{4}x}\cdot e^{\frac{-4y+7x}{52}}\left(\sin\left(\frac{4y-7x}{13}\right)+2\right)$$

$$= -2 + e^{(-y+5x)/13}\left[\sin\left(\frac{4y-7x}{13}\right)+2\right]$$