Math 3326 Quiz #1

SPRING SEMESTER 2009

Name SOLUTIONS

1. Suppose that $f(x)$ and $g(y)$ are arbitrary differentiable functions of one variable. Verify that $u(x,y) = f(x)g(y)$ is a solution of the second-order (nonlinear) PDE

$$uu_{xy} - u_x u_y = 0.$$

$u(x,y) = f(x)g(y)$

$u_x(x,y) = f'(x)g(y)$

$u_y(x,y) = f(x)g'(y)$

$u_{xy}(x,y) = f'(x)g'(y)$

$\therefore\ uu_{xy} - u_x u_y$

$= f(x)g(y)f'(x)g'(y) - f'(x)g(y)f(x)g'(y)$

$= 0.$

2. Consider the first-order PDE $u_x + u_y = 1$.

   (a) Find the characteristic equation and the transformed equation associated with this PDE. From this, find the general solution to this PDE. Simplify your answer as much as possible.

   The characteristic equation is $\frac{dy}{dx} = 1$ so $y = x + C$.
   Let $\eta(x,y) = y - x$ & $\xi(x,y) = x$ so $J = \begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1 \neq 0$

   The transformed equation is $w_\xi = 1$ so $w(\xi,\eta) = \xi + f(\eta)$

   Hence the general solution to $u_x + u_y = 1$ is

   $$u(x,y) = x + f(y-x)$$

   (b) Now find the solution to this same PDE when we additionally require that $u(0,y) = y$. Again, simplify your solution as much as possible.

   Given that $u(x,y) = x + f(y-x)$ and we want $y = u(0,y)$, we see that

   $$y = u(0,y) = f(y)$$

   Hence $u(x,y) = x + f(y-x) = x + (y-x) = y$

   ie $u(x,y) = y$ is the solution to $\begin{cases} u_x + u_y = 1 \\ u(0,y) = y. \end{cases}$