Math 3326 Quiz #1

FALL SEMESTER 2008

Name SOLUTIONS

1. Verify that

$$u(x, y) = e^{-4x} f(2x - 3y)$$

is a solution of the first-order pde

$$3u_x + 2u_y + 12u = 0.$$

$$u_x = -4e^{-4x} f(2x-3y) + 2e^{-4x} f'(2x-3y)$$
$$u_y = -3e^{-4x} f'(2x-3y)$$
$$\therefore\ 3u_x + 2u_y + 12u = -12e^{-4x} f(2x-3y) + 6e^{-4x} f'(2x-3y)$$
$$-6e^{-4x} f'(2x-3y) + 12e^{-4x} f(2x-3y)$$
$$= 0.$$

2. For the following differential equation, find the transformed equation, the characteristic equation, and the general solution (in terms of x and y):

$$u_x + yu_y + xu = 0.$$

- $\xi = x$, $\eta = \eta(x,y)$
- Transformed equation: $w_\xi + xw = 0$

or $\boxed{w_\xi + \xi w = 0}$

Multiply by $e^{\int \xi d\xi} = e^{\xi^2/2}$: $(e^{\xi^2/2} w)_\xi = 0$

so $e^{\xi^2/2} w = g(\eta)$ and $w = e^{-\xi^2/2} g(\eta)$

- Characteristic equation:

$\boxed{\frac{dy}{dx} = y} \Rightarrow \frac{dy}{y} = dx$ so $\ln|y| = x + c$

$\Rightarrow y = Ae^x$ or

$\Rightarrow ye^{-x} = A$

Let $\eta(x,y) = ye^{-x}$ or $\eta(x,y) = \ln|y| - x$

Then $\boxed{u(x,y) = e^{-x^2/2} g(ye^{-x})}$ is the general solution.

or $= \boxed{e^{-x^2/2} g(\ln|y| - x)}$