Math 3326
Fall Semester 2008
Problem Set #9

1. Solve the Cauchy wave problem

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \quad (0 \le x \le L; -\infty < t < \infty) \\ u(0,t) &= u(L,t) = 0 \\ u(x,0) &= f(x);\ u_t(x,0) = g(x) \end{aligned}$$

in the following cases:

(a) $f(x) = 3\sin\left(\frac{\pi x}{L}\right) - \sin\left(\frac{4\pi x}{L}\right)$, $g(x) = \frac{1}{2}\sin\left(\frac{2\pi x}{L}\right)$;

(b) $f(x) = \sin^3\left(\frac{\pi x}{L}\right)$, $g(x) = 0$ (Hint: use a trigonometric identity);

(c) $f(x) = 0$, $g(x) = \sin\left(\frac{\pi x}{L}\right)\cos^2\left(\frac{\pi x}{L}\right)$ (Hint: use a trigonometric identity);

(d) $f(x) = \sin^3\left(\frac{\pi x}{L}\right)$, $g(x) = \sin\left(\frac{\pi x}{L}\right)\cos^2\left(\frac{\pi x}{L}\right)$ (Hint: use your answers from (b) and (c) in a clever way!).

2. Solve the Cauchy wave problem

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \quad (0 \le x \le L; -\infty < t < \infty) \\ u(0,t) &= u(L,t) = 0 \\ u(x,0) &= f(x);\ u_t(x,0) = g(x) \end{aligned}$$

in the following cases:

(a) $f(x) = 1 - \cos x$, $g(x) = 0$, $c = 3$, $L = 2\pi$;

(b) $f(x) = x(1-x)$, $g(x) = 0$, $c = 6$, $L = 1$.

(c) $f(x) = x^2(2-x)$, $g(x) = x^2$, $c = 5$, $L = 2$.

3. Use the separation of variables technique, like we did in class to solve the wave equation, to solve the *telegraph equation*

$$u_{tt} + Au_t + Bu = c^2 u_{xx} \quad (0 < x < L,\ t > 0)$$

in which A, B, and c are positive constants. The boundary conditions are

$$u(0,t) = u(L,t) = 0 \text{ for } t > 0$$

and the initial conditions are

$$u(x,0) = f(x) \text{ and } u_t(x,0) = 0 \quad (0 < x < L).$$

Assume that $A^2L^2 < 4(BL^2 + c^2\pi^2)$.