

Math 3326  
Fall Semester 2008  
Problem Set #7

1. In each of the following problems, find the canonical form of the following elliptic PDEs:

(a)  $u_{xx} + 2u_{xy} + 5u_{yy} + u_y = 0$

(b)  $3u_{xx} - 2u_{xy} + 2u_{yy} = 0$

(c)  $4u_{xx} + u_{xy} + u_{yy} + u_x - xu_y + yu = 0$

(d)  $2u_{xx} - 2u_{xy} + u_{yy} + \cos(y)u_x = 0$ .

2. Let  $u$  satisfy the elliptic equation  $u_{xx} + u_{yy} = 0$ . Show that  $u_x$ ,  $u_y$ ,  $u_{xx}$ ,  $u_{xy}$ , and  $u_{yy}$  are also solutions. Verify this conclusion by direct calculation for the solution

$$u(x, y) = \sin(x) \cosh(y).$$