

Math 3326  
Fall Semester 2008  
Problem Set #6

1. In each of the following problems, find a canonical form of the following PDEs and use this to find a solution in terms of two arbitrary, twice differentiable functions:

(a)  $u_{xx} + 6u_{xy} + 9u_{yy} = 0$

(b)  $u_{xx} - 4u_{xy} + 4u_{yy} = 0$

(c)  $25u_{xx} + 20u_{xy} + 4u_{yy} = 0$

(d)  $9u_{xx} + 12u_{xy} + 4u_{yy} = 0$ .

2. Determine the characteristics of the canonical form of the parabolic equation.
3. Verify that the PDE  $u_y - ku_{xx} = 0$  is parabolic on the region  $\mathcal{D} = \mathbb{R}^2$  for any positive constant  $k$ . Suppose that  $u$  is a solution of this PDE. Show that  $u_x$ ,  $u_y$ ,  $u_{xx}$ ,  $u_{xy}$ , and  $u_{yy}$  are also solutions. Verify this conclusion by direct calculation for the solution

$$u(x, y) = e^{-\alpha^2 \pi^2 y} \cos(\alpha \pi x),$$

in which  $\alpha$  can be any nonzero real number.