

Math 3326  
Fall Semester 2008  
Problem Set #5

1. In each of the following problems, find the canonical form of the following PDEs and use this to find a solution in terms of two arbitrary, twice differentiable functions:

(a)  $u_{xx} + 2u_{xy} - 3u_{yy} = 0$

(b)  $u_{xx} + 4u_{xy} + u_{yy} = 0$

(c)  $2u_{xx} + 2u_{xy} - 5u_{yy} = 0$

(d)  $3u_{xx} + 6u_{xy} + 2u_{yy} = 0.$

2. Consider the transformation to canonical form in terms of the variables  $\xi, \eta$  in the hyperbolic case. Let  $\hat{\xi} = \xi + \eta$ ,  $\hat{\eta} = \xi - \eta$ , and  $\hat{w}(\hat{\xi}, \hat{\eta}) = w(\xi, \eta)$ , to obtain

$$\hat{w}_{\hat{\xi}\hat{\xi}} - \hat{w}_{\hat{\eta}\hat{\eta}} + \psi(\hat{\xi}, \hat{\eta}, \hat{w}, \hat{w}_{\hat{\xi}}, \hat{w}_{\hat{\eta}}) = 0.$$

This is another canonical form commonly used for the hyperbolic second order linear partial differential equation. Obtain this canonical form for the following hyperbolic equations:

(a)  $u_{xx} - 8u_{xy} + 7u_{yy} + u_x = 0$

(b)  $u_{xx} - 10u_{xy} + 16u_{yy} - xu_x = 0.$