

Math 3326
Fall Semester 2008
Problem Set #12

1. Find the Fourier series for each of the following functions:

$$(a) f(x) = \begin{cases} 1 & -\pi \leq x < 0 \\ 2 & x = 0 \\ 3 & 0 < x \leq \pi. \end{cases}$$

(i) What is the value FS $f(x) |_{x=0}$? Deduce, if possible, an infinite series identity from this.

(ii) What is the value FS $f(x) |_{x=-\pi}$? Deduce, if possible, an infinite series identity from this.

$$(b) f(x) = \begin{cases} x & -\pi \leq x < 0 \\ 2 & 0 \leq x \leq \pi. \end{cases}$$

(i) What is the value of FS $f(x) |_{x=0}$? Deduce, if possible, an infinite series identity from this.

(ii) What is the value of FS $f(x) |_{x=\frac{\pi}{2}}$? Deduce, if possible, an infinite series identity from this.

$$(c) f(x) = \begin{cases} 1 & -\pi \leq x < 0 \\ \sin x & 0 \leq x \leq \pi. \end{cases}$$

(i) What is the value of FS $f(x) |_{x=-\frac{\pi}{2}}$?

(ii) What is the value of FS $f(x) |_{x=\frac{\pi}{2}}$?

2. Find the Fourier sine series of each of the following functions:

(a) $f(x) = 1$ on $[0, \pi]$.

(b) $f(x) = \cos x$ on $[0, \pi]$.

(c) $f(x) = \sin x$ on $[0, \pi]$.

3. Find the Fourier cosine series of each of the following functions:

(a) $f(x) = 1$ on $[0, \pi]$.

(b) $f(x) = x$ on $[0, \pi]$.

(c) $f(x) = \sin x$ on $[0, \pi]$.