

Math 3326
Fall Semester 2008
Problem Set #11

1. Find the Fourier series of the following functions without computing any integrals:

(a) $f(x) = \cos^2(\pi x) \sin^2(\pi x)$

(b) $f(x) = \sin x [\sin x + \cos x]^2$.

2. Suppose

$$f(x) = \sum_{n=1}^N a_n \cos\left(\frac{n\pi x}{L}\right)$$

and

$$g(x) = \sum_{n=1}^N a'_n \cos\left(\frac{n\pi x}{L}\right)$$

for some constants a_n and a'_n . Show that

$$\int_{-L}^L f(x)g(x)dx = (f, g) = L \sum_{n=1}^N a_n a'_n.$$

3. Compute the Fourier series for $f(x) = x^2$ for all $x \in [-L, L]$. Then

(a) Deduce that $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} = \frac{\pi^2}{12}$.

(b) Deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

(c) What is the value of $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$?