Math 3326
Fall Semester 2008
Problem Set #10

1. In each of the following problems, solve

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u(x,0) &= \varphi(x),\ u_t(x,0) = \psi(x) \end{aligned}$$

when

(a) $c = 7$, $\varphi(x) = \cos 3x$, $\psi(x) = x$;

(b) $c = 4$, $\varphi(x) = x^2$, $\psi(x) = \sin 2x$;

(c) $c = 1$, $\varphi(x) = x + 2$, $\psi(x) = e^x$;

(d) $c = 2$, $\varphi(x) = \cosh x$, $\psi(x) = 2x$.

2. Let w be a solution of the problem

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u(x,0) &= \varphi(x),\ u_t(x,0) = 0. \end{aligned}$$

Let v be a solution of

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u(x,0) &= 0,\ u_t(x,0) = \psi(x). \end{aligned}$$

Prove that $w + v$ is a solution of

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u(x,0) &= \varphi(x),\ u_t(x,0) = \psi(x). \end{aligned}$$

3. Verify that

$$u(x,t) = \frac{1}{2c} \int_0^t \int_{x-c(t-u)}^{x+c(t-u)} f(\xi, u)\, d\xi\, du$$

is a solution of

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u(0,t) &= 0 \\ u_t(x,0) &= 0. \end{aligned}$$