Math 3326
Fall Semester 2008
Problem Set #1

1. Show that if p is a continuously differentiable function of one variable, the first-order partial differential equation

$$u_t = p(u)u_x$$

has a solution implicitly defined by

$$u(x,t) = \varphi(x + p(u)t),$$

in which φ can be any continuously differentiable function of one variable. Use this idea to determines (perhaps implicitly) a solution of each of the following equations:

   (a) $u_t = ku_x$ (with k being a nonzero constant)

   (b) $u_t = uu_x$

   (c) $u_t = \cos(u)u_x$

   (d) $u_t = u\sin(u)u_x$.

2. Show that $u(x,y) = \ln((x-x_0)^2 + (y-y_0)^2)$ satisfies $u_{xx} + u_{yy} = 0$ for all pairs (x,y) of real numbers except (x_0,y_0).

3. Let k be a positive constant. Let

$$u(x,t) = \frac{1}{2\sqrt{\pi kt}} \int_{-\infty}^{\infty} e^{-(x-\xi)^2/4kt} f(\xi)d\xi,$$

in which f is continuous on the real line. Show that $u_t = ku_{xx}$ for $-\infty < x < \infty$, $t > 0$. Also determine $u(x,t)$ when $f(x) = 1$ for all real x. Hint: Use a change of variable and the standard result that

$$\int_{-\infty}^{\infty} e^{-w^2} dw = \sqrt{\pi}.$$