**Math 3325 Quiz #8**
FALL SEMESTER 2009

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Name SOLUTIONS.

1. Find a particular solution to the non-homogeneous differential equation

$$y'' - 3y' + 2y = (1 + e^{-x})^{-1}.$$

The characteristic equation associated with $y''-3y'+2y=0$ is $\alpha^2-3\alpha+2=0$ or $(\alpha-2)(\alpha-1)=0$.

Consequently, we take $y_1(x)=e^{2x}$ and $y_2(x)=e^{x}$.

Then $W(y_1,y_2)(x)=\begin{vmatrix} e^{2x} & e^{x} \\ 2e^{2x} & e^{x}\end{vmatrix} = e^{3x}-2e^{3x} = -e^{3x}$

Hence

$$c_1(x) = -\int \frac{e^{x}\,dx}{(1+e^{-x})(-e^{3x})} = \int \frac{e^{-2x}}{1+e^{-x}}dx$$
$$= -1-e^{-x}+\ln(1+e^{-x})$$

let $u = 1+e^{-x}$
$\Rightarrow du = -e^{-x}dx$
& $e^{-x} = u-1$
$\therefore \int \frac{e^{-2x}dx}{1+e^{-x}}$
$= -\int \frac{u-1}{u}du$
$= -u+\ln u$
$= -1-e^{-x}+\ln(1+e^{-x})$

$$c_2(x) = \int \frac{e^{2x}\,dx}{(1+e^{-x})(-e^{3x})}$$
$$= -\int \frac{e^{-x}}{1+e^{-x}}dx$$
$$= \ln(1+e^{-x})$$

$u = 1+e^{-x}$
$du = -e^{-x}dx$
$\therefore -\int \frac{e^{-x}dx}{1+e^{-x}}$
$= \int \frac{du}{u} = \ln u = \ln(1+e^{-x})$

$$\therefore\ y_p(x) = c_1(x)y_1(x)+c_2(x)y_2(x)$$
$$= (-1-e^{-x}+\ln(1+e^{-x}))e^{2x} + e^{x}\ln(1+e^{-x})$$
$$= -e^{2x}-e^{x}+(e^{x}+e^{2x})\ln(1+e^{-x})$$

Since $-e^{2x}-e^{x}$ is a solution of $y''-3y'+2y=0$ we can take

$$y_p(x) = (e^{x}+e^{2x})\ln(1+e^{-x})$$