

1. A cup of liquid is put into a microwave oven and heated to 180F. The cup is then taken out and put in a room of constant temperature of 70F. After 2 minutes the liquid has cooled to 160F. What will the temperature, to the nearest degree, of the liquid be at 10 minutes?

Let $T(t)$ be the temperature of the liquid at time t .

Then, we must solve the IVP
$$\begin{cases} \frac{dT}{dt} = k(T-70) \\ T(0) = 180 \end{cases}$$

$$\frac{dT}{dt} = k(T-70) \Rightarrow \frac{dT}{T-70} = k dt \Rightarrow \ln|T-70| = kt + c$$

$$\Rightarrow T(t) = 70 + Ae^{kt}$$

$$180 = T(0) = 70 + A \Rightarrow A = 110 \text{ so } \boxed{T(t) = 70 + 110e^{kt}}$$

$$\text{Moreover, } 160 = T(2) = 70 + 110e^{2k}$$

$$\Rightarrow \frac{90}{110} = e^{2k} \Rightarrow 2k = \ln\left(\frac{9}{11}\right) \text{ so } k = \frac{1}{2} \ln\left(\frac{9}{11}\right)$$

$$\S T(t) = 70 + 110e^{\frac{t}{2} \ln\left(\frac{9}{11}\right)}$$

Now we find $T(10)$:

$$T(10) = 70 + 110e^{\frac{10}{2} \ln\left(\frac{9}{11}\right)}$$

$$= 70 + 110e^{5 \ln\left(\frac{9}{11}\right)}$$

$$\approx 110.33 \text{ F}$$