Math 3325 Quiz #3

FALL SEMESTER 2009

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Name SOLUTIONS

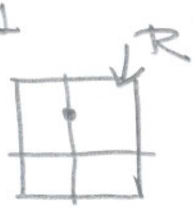
1. Consider the IVP:

$$\frac{dy}{dx} + y = e^{-x} + x - 1$$
$$y(0) = 1.$$

(a) Using the Existence/Uniqueness Theorem, argue that there is a unique solution to this problem. State the conditions precisely and be sure to draw an acceptable rectangle R.

Let $f(x,y) = e^{-x} + x - 1 - y$ so $\frac{\partial f}{\partial y}(x,y) = -1$

Both of these functions are continuous in any rectangle R containing (0,1) inside of it so, by E/U, there is a unique solution to this IVP.



(b) Find this unique solution explicitly.

Multiply $\frac{dy}{dx} + y = e^{-x} + x - 1$ by $e^{\int dx} = e^x$:

We get

$$(e^x y)' = 1 + xe^x - e^x$$

Integrate:

$$e^x y(x) = x + (xe^x - e^x) - e^x + C$$
$$\Rightarrow y(x) = xe^{-x} + x - 1 - 1 + Ce^{-x}$$
$$= xe^{-x} + x - 2 + Ce^{-x}$$

$1 = y(0) = -2 + C$ so $C = 3$

$$\Rightarrow \boxed{y(x) = xe^{-x} + x - 2 + 3e^{-x}}$$