Math 3325 Quiz #2
FALL SEMESTER 2009

Lance L. Littlejohn

Name SOLUTIONS

1. Consider the IVP

$$xy' = -2y$$
$$y(1) = 0.$$

(a) Prove, using the E/U theorem that this IVP has a unique solution. Write clear, precise statements to support your proof (and an explicit picture would help too).

Here, $f(x,y) = \frac{-2y}{x}$ so $\frac{\partial f}{\partial y}(x,y) = \frac{-2}{x}$

Both of these functions are continuous in any rectangle R containing the initial point (1,0) that avoids the y-axis (ie, where $x=0$).

$\therefore$ there is a unique solution to this problem.

(b) Solve this IVP for the explicit solutions $y(x)$.

$$xy' = -2y \Rightarrow \frac{dy}{y} = \frac{-2\,dx}{x} \Rightarrow \ln|y| = -2\ln|x| + K$$
$$\Rightarrow y = cx^{-2}$$

When $x=1$, $y=0 \Rightarrow c=0$ ie, $y \equiv 0$ is the solution

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