Math 3325 Quiz #1

FALL SEMESTER 2009

Lance L. Littlejohn

Name SOLUTIONS

1. Verify whether or not the relation $x^2y+xy^2=xy$ defines an implicit solution to the differential equation

$$\frac{dy}{dx}=\frac{y-2xy-y^2}{x^2+2xy-x}.$$

Differentiate $x^2y+xy^2=xy$ implicitly with respect to x:

$$2xy+x^2y'+y^2+2xyy'=y+xy'$$

Gather up the y' terms: $(x^2+2xy-x)y'=y-2xy-y^2$

So that $\frac{dy}{dx}=\frac{y-2xy-y^2}{x^2+2xy-x}$; thus, yes the above relation does define a solution.

2. Verify whether or not the relation $e^{xy}+\sin(xy)=0$ defines an implicit solution to the differential equation

$$\frac{dy}{dx}=\frac{ye^{xy}}{xe^{xy}+\cos(xy)}.$$

Same procedure as above: $(xy)'e^{xy}+(xy)'\cos(xy)=0$

so $e^{xy}[y+xy']+\cos(xy)[y+xy']=0$

Gather up the y' terms: $y'[xe^{xy}+x\cos xy]=-ye^{xy}-y\cos xy$

So $\frac{dy}{dx}=\frac{-ye^{xy}-y\cos xy}{xe^{xy}+x\cos xy}$; this time the answer is 'no'.

3. Show that $y(x)=c_1\cos x+c_2\sin x+c_3\bar{e}^{x}$ is a solution of the differential equation

$$y'''+y''+y'+y=0.$$

$$y'(x)=-c_1\sin x+c_2\cos x-c_3\bar{e}^{x}$$
$$y''(x)=-c_1\cos x-c_2\sin x+c_3\bar{e}^{x}$$
$$y'''(x)=c_1\sin x-c_2\cos x-c_3\bar{e}^{x}$$

$$\therefore\ y'''+y''+y'+y=(\cancel{c_1\sin x}-\cancel{c_2\cos x}-\cancel{c_3\bar{e}^{x}})+(-\cancel{c_1\cos x}-\cancel{c_2\sin x}+\cancel{c_3\bar{e}^{x}})$$
$$+(-\cancel{c_1\sin x}+\cancel{c_2\cos x}-\cancel{c_3\bar{e}^{x}})+(\cancel{c_1\cos x}+\cancel{c_2\sin x}+\cancel{c_3\bar{e}^{x}})$$
$$=0\ \checkmark$$