

# Math 2311 Quiz #9

SPRING SEMESTER 2008

Name SOLUTIONS

1. Let  $u = (2, 3, 1)$ ,  $W = \text{span}\{u\}$ , and  $x = (1, 1, 5)$ .

(a) Find  $\text{proj}_W x$ , the orthogonal projection of  $x$  onto  $W$ .

$$\text{proj}_W x = \frac{x \cdot u}{u \cdot u} u = \frac{10}{14} u = \frac{5}{7} (2, 3, 1) \quad \left( \text{or } \left( \frac{10}{7}, \frac{15}{7}, \frac{5}{7} \right) \right)$$

(b) Write  $x$  as the sum of two vectors, one coming from  $W$  and the other from  $W^\perp$ . Simplify your vectors as much as possible.

$$\begin{aligned} (1, 1, 5) &= \text{proj}_W x + \left( (1, 1, 5) - \text{proj}_W x \right) \\ &= \left( \frac{10}{7}, \frac{15}{7}, \frac{5}{7} \right) + \left( -\frac{3}{7}, -\frac{8}{7}, \frac{30}{7} \right) \end{aligned}$$

(c) Find the distance between  $x$  and  $W$ . Simplify your answer as much as possible.

$$\begin{aligned} d(x, W) &= \left\| (1, 1, 5) - \text{proj}_W x \right\| \\ &= \left\| \left( -\frac{3}{7}, -\frac{8}{7}, \frac{30}{7} \right) \right\| \\ &= \frac{\sqrt{9 + 64 + 900}}{7} = \frac{\sqrt{973}}{7} \end{aligned}$$