

Math 2311 Quiz #8

SPRING SEMESTER 2008

Name SOLUTIONS

1. Let

$$W = \{(x, y, z, w) \mid -x + y + 3z + w = 0; 2x - y + z = -2w\}.$$

(a) Indicate, with a solid reason, why W is a subspace of \mathbb{R}^4 .

$W = \text{null} \begin{pmatrix} -1 & 1 & 3 & 1 \\ 2 & -1 & 1 & 2 \end{pmatrix}$; since nullspaces of matrices are subspaces, this explains why W is a subspace of \mathbb{R}^4 .

(b) Find a basis for W .

$$\begin{pmatrix} -1 & 1 & 3 & 1 \\ 2 & -1 & 1 & 2 \end{pmatrix} \xrightarrow{2R_1+R_2} \begin{pmatrix} -1 & 1 & 3 & 1 \\ 0 & 1 & 7 & 4 \end{pmatrix} \xrightarrow{-R_2+R_1} \begin{pmatrix} -1 & 0 & -4 & -3 \\ 0 & 1 & 7 & 4 \end{pmatrix}$$

$$\begin{aligned} \text{Hence } -x - 4z - 3w &= 0 & \text{so } x &= -4z - 3w \\ y + 7z + 4w &= 0 & y &= -7z - 4w \end{aligned}$$

\therefore every vector in W has the form

$$(x, y, z, w) = (-4z - 3w, -7z - 4w, z, w) = z(-4, -7, 1, 0) + w(-3, -4, 0, 1)$$

\therefore a basis for W is $\{(-4, -7, 1, 0), (-3, -4, 0, 1)\}$

(c) Find a basis for W^\perp .

A basis for $W^\perp = \text{Row}(A)$ is $\{(-1, 1, 3, 1), (2, -1, 1, 2)\}$

or $\{(-1, 0, -4, -3), (0, 1, 7, 4)\}$.