Math 2311 Quiz #7

FALL SEMESTER 2007

Name SOLUTIONS

1. Find a basis for

$$W = \{(x_1, x_2, x_3, x_4) \mid x_1 - 2x_2 = 3x_3; x_1 - x_2 + x_3 - x_4 = 0\}.$$

$W = \text{null}\begin{pmatrix} 1 & -2 & -3 & 0 \\ 1 & -1 & 1 & -1 \end{pmatrix}$ $\begin{pmatrix} 1 & -2 & -3 & 0 \\ 1 & -1 & 1 & -1 \end{pmatrix} \xrightarrow{-R_1+R_2} \begin{pmatrix} 1 & -2 & -3 & 0 \\ 0 & 1 & 4 & -1 \end{pmatrix}$

$\xrightarrow{2R_2+R_1} \begin{pmatrix} 1 & 0 & 5 & -2 \\ 0 & 1 & 4 & -1 \end{pmatrix}$ Hence $x_1 = -5x_3 + 2x_4$

$x_2 = -4x_3 + x_4$

So every vector in W has the form (x_1, x_2, x_3, x_4)

$= (-5x_3+2x_4, -4x_3+x_4, x_3, x_4) = x_3(-5,-4,1,0) + x_4(2,1,0,1)$

∴ A basis for W is $\{(-5,-4,1,0), (2,1,0,1)\}$.

2. Find a basis

$$W = \{p \in \mathcal{P}_4 \mid p'(0) = 2p'(1)\}.$$

Let $p \in P_4$ so $p(x) = ax^3 + bx^2 + cx + d$

Then $p'(x) = 3ax^2 + 2bx + c$.

Hence $p \in W \iff p'(0) = 2p'(1)$

$\iff c = 6a + 4b + 2c$

$\iff 6a + 4b + c = 0$

$\iff c = -6a - 4b$

Hence $p \in W \iff p(x) = ax^3 + bx^2 + (-6a-4b)x + d$

$= a(x^3 - 6x) + b(x^2 - 4x) + d \cdot 1$

∴ a basis for W is

$\{x^3 - 6x, x^2 - 4x, 1\}$.