Math 2311 Quiz #6

SPRING SEMESTER 2008

Name SOLUTIONS

1. Let $A = \begin{pmatrix} -1 & 2 & 1 & 0 \\ 2 & 1 & 3 & -2 \\ 3 & 4 & 7 & -4 \\ 0 & 5 & 5 & -2 \end{pmatrix}$. Find a basis for null($A$).

$$A \xrightarrow[3R_1+R_3]{2R_1+R_2} \begin{pmatrix} -1 & 2 & 1 & 0 \\ 0 & 5 & 5 & -2 \\ 0 & 10 & 10 & -4 \\ 0 & 5 & 5 & -2 \end{pmatrix} \xrightarrow[\substack{-2R_2+R_3 \\ -R_2+R_3}]{5R_1} \begin{pmatrix} -5 & 10 & 5 & 0 \\ 0 & 5 & 5 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{-2R_2+R_1} \begin{pmatrix} -5 & 0 & -5 & 4 \\ 0 & 5 & 5 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow[\frac{1}{5}R_2]{-\frac{1}{5}R_1} \begin{pmatrix} 1 & 0 & 1 & -4/5 \\ 0 & 1 & 1 & -2/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

ie/ x_1, x_2: basic $\quad x_1 + x_3 - \frac{4}{5}x_4 = 0$

x_3, x_4: free $\quad x_2 + x_3 - \frac{2}{5}x_4 = 0$

So $x_1 = -x_3 + \frac{4}{5}x_4$ & $x_2 = -x_3 + \frac{2}{5}x_4$

Every vector (x_1, x_2, x_3, x_4) has the form $(-x_3 + \frac{4}{5}x_4, -x_3 + \frac{2}{5}x_4, x_3, x_4)$

$= x_3(-1,-1,1,0) + x_4(\frac{4}{5}, \frac{2}{5}, 0, 1)$ so a basis for null(A) is $\{(-1,-1,1,0), (4,2,0,5)\}$.

2. Let $W = \{p \in \mathcal{P}_4 \mid p(0) - 2p'(0) = 3p''(0)\}$; then W is a subspace of $\mathcal{P}_4$. Find a basis for W.

$p(x) = ax^3 + bx^2 + cx + d \in W \iff p(0) - 2p'(0) - 3p''(0) = 0$

Now $p'(x) = 3ax^2 + 2bx + c$

$p''(x) = 6ax + 2b$

so $0 = p(0) - 2p'(0) - 3p''(0) = d - 2c - 6b$ so $d = 2c + 6b$

$\therefore$ $p(x) = ax^3 + bx^2 + cx + 2c + 6b$

$= ax^3 + b(x^2+6) + c(x+2)$

So a basis for W is $\{x^3, x^2+6, x+2\}$.