

Math 2311 Quiz #5

FALL SEMESTER 2007

Name SOLUTIONS

1. Let $A = \begin{pmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -3 & 1 \\ 2 & -2 & -1 & 5 & 3 \\ -2 & 2 & 2 & -8 & -2 \end{pmatrix} \xrightarrow{\substack{-2R_1+R_3 \\ 2R_1+R_4}} \begin{pmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 2 & -6 & 2 \end{pmatrix}$

(a) Put A into RREF form.

$$\xrightarrow{\substack{R_2+R_3 \\ -2R_2+R_4}} \begin{pmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(b) Find a basis for $\text{row}(A)$. What is $\dim(\text{row}(A))$?

$$\{(1, -1, 0, 1, 2), (0, 0, 1, -3, 1)\} \quad \dim = 2$$

(c) Find a basis for $\text{col}(A)$. What is $\dim(\text{col}(A))$?

$$\{(1, 0, 2, -2), (0, 1, -1, 2)\} \quad \dim = 2$$

(d) Find a basis for $\text{null}(A)$. What is $\dim(\text{null}(A))$?

$$x_1 - x_2 + x_4 + 2x_5 = 0$$

$$x_3 - 3x_4 + x_5 = 0$$

$$\left. \begin{matrix} x_1 \\ x_3 \end{matrix} \right\} \text{basic} \quad \left. \begin{matrix} x_2 \\ x_4 \\ x_5 \end{matrix} \right\} \text{free}$$

$$\text{So } x_1 = x_2 - x_4 - 2x_5$$

$$x_3 = 3x_4 - x_5$$

Every solution has the form $(x_1, x_2, x_3, x_4, x_5)$

$$= (x_2 - x_4 - 2x_5, x_2, 3x_4 - x_5, x_4, x_5)$$

$$= x_2(1, 1, 0, 0, 0) + x_4(-1, 0, 3, 1, 0)$$

$$+ x_5(-2, 0, -1, 0, 1)$$

∴ a basis for $\text{null}(A)$ is

$$\{(1, 1, 0, 0, 0), (-1, 0, 3, 1, 0), (-2, 0, -1, 0, 1)\}$$