Math 2311 Quiz #4

SPRING SEMESTER 2008

Name SOLUTIONS

1. Let $A = \begin{pmatrix} 1 & -2 & 0 & 1 & 1 \\ 0 & 0 & 2 & 4 & 8 \\ 3 & -6 & 2 & 4 & 2 \\ 2 & -4 & 2 & 3 & 1 \end{pmatrix}$.

(a) Put A into RREF form.

$$A \xrightarrow[-2R_1+R_2]{-3R_1+R_3} \begin{pmatrix} 1 & -2 & 0 & 1 & 1 \\ 0 & 0 & 2 & 4 & 8 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 2 & 1 & -1 \end{pmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{pmatrix} 1 & -2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 2 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow[-2R_2+R_4]{-2R_2+R_3} \begin{pmatrix} 1 & -2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & -3 & -9 \\ 0 & 0 & 0 & -3 & -9 \end{pmatrix} \xrightarrow[-\frac{1}{3}R_3]{-R_3+R_4} \begin{pmatrix} 1 & -2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow[-2R_3+R_1]{-R_3+R_1} \begin{pmatrix} 1 & -2 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(b) Find a basis for row(A). What is dim(row(A))?

$\{(1,-2,0,0,-2), (0,0,1,0,-2), (0,0,0,1,3)\}$

(c) Find a basis for col(A). What is dim(col(A))?

$\{(1,0,3,2), (0,2,2,2), (1,4,4,3)\}$

(d) Find a basis for null(A). What is dim(null(A))?

$x - 2y - 2t = 0$
$z - 2t = 0$
$w + 3t = 0$

$\Rightarrow x = 2y + 2t$
$z = 2t$
$w = -3t$

Basis variables: x, z, w
Free variables: y, t

Every solution is of the form

$(x, y, z, w, t) = (2y+2t, y, 2t, -3t, t)$
$= y(2,1,0,0,0) + t(2,0,2,-3,1)$

A basis for null(A) is $\{(2,1,0,0,0), (2,0,2,-3,1)\}$.