

Math 2311 Quiz #4

FALL SEMESTER 2007

Name SOLUTIONS

1. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ by the linear transformation defined by

$$T(x, y, z) = (2x - y + z, x + 3y + 2z).$$

(a) Find a 2×3 matrix A such that $T(x, y, z) = A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

$$\text{Let } A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 3 & 2 \end{pmatrix}$$

(b) Is there a solution (x, y, z) to $T(x, y, z) = (1, 2)$. If there more than one solution? If so, find a parametric form for all solutions to this equation.

We solve $T(x, y, z) = (1, 2)$ by putting the augmented matrix $\begin{pmatrix} 2 & -1 & 1 & 1 \\ 1 & 3 & 2 & 2 \end{pmatrix}$ into RREF

$$\begin{pmatrix} 2 & -1 & 1 & 1 \\ 1 & 3 & 2 & 2 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 3 & 2 & 2 \\ 2 & -1 & 1 & 1 \end{pmatrix} \xrightarrow{-2R_1 + R_2} \begin{pmatrix} 1 & 3 & 2 & 2 \\ 0 & -7 & -3 & -3 \end{pmatrix}$$

$$\xrightarrow{7R_2} \begin{pmatrix} 7 & 21 & 14 & 14 \\ 0 & -7 & -3 & -3 \end{pmatrix} \xrightarrow{3R_2 + R_1} \begin{pmatrix} 7 & 0 & 5 & 5 \\ 0 & -7 & -3 & -3 \end{pmatrix} \quad \text{ie/ } \begin{aligned} x &= \frac{5}{7} - \frac{5z}{7} \\ y &= \frac{3}{7} - \frac{3z}{7} \end{aligned}$$
$$\xrightarrow{\begin{matrix} 1/7 R_1 \\ -1/7 R_2 \end{matrix}} \begin{pmatrix} 1 & 0 & 5/7 & 5/7 \\ 0 & 1 & 3/7 & 3/7 \end{pmatrix}$$

Every solution (x, y, z) to $T(x, y, z) = (1, 2)$ has the form

$$(x, y, z) = \left(\frac{5}{7} - \frac{5z}{7}, \frac{3}{7} - \frac{3z}{7}, z \right) = z \left(-\frac{5}{7}, -\frac{3}{7}, 1 \right) + \left(\frac{5}{7}, \frac{3}{7}, 0 \right).$$