Math 2311 Quiz #3

FALL SEMESTER 2007

Name SOLUTIONS

1. Express, if possible, if $\mathbf{x} = (1, -2, 5)$ as a linear combination of $\mathbf{x}_1 = (1, 1, 1)$, $\mathbf{x}_2 = (1, 2, 3)$, and $\mathbf{x}_3 = (2, -1, 1)$.

$$\begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 2 & -1 & -2 \\ 1 & 3 & 1 & 5 \end{pmatrix} \xrightarrow[-R_1+R_3]{-R_1+R_2} \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 2 & -1 & 4 \end{pmatrix} \xrightarrow[-2R_2+R_3]{-R_2+R_1} \begin{pmatrix} 1 & 0 & 5 & 4 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 5 & 10 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{5}R_3} \begin{pmatrix} 1 & 0 & 5 & 4 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[-5R_3+R_1]{3R_3+R_2} \begin{pmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

$$\therefore\ (1,-2,5) = -6(1,1,1) + 3(1,2,3) + 2(2,-1,1)$$

2. Determine if each of the following set of vectors is linearly dependent or linearly independent in $\mathbb{R}^3$:

(a) $x_1 = (1, 2, 5)$, $x_2 = (2, 5, 1)$, $x_3 = (1, 5, 2)$

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 5 \\ 5 & 1 & 2 \end{pmatrix} \xrightarrow[-5R_1+R_3]{-2R_1+R_2} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & -9 & -3 \end{pmatrix} \xrightarrow[9R_2+R_3]{-2R_2+R_1} \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & 3 \\ 0 & 0 & 24 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{24}R_3} \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[5R_3+R_1]{-3R_3+R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \therefore \text{ LI vectors}$$

(b) $x_1 = (1, 2, 3)$, $x_2 = (-1, 3, -2)$, $x_3 = (2, 9, 7)$.

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & 9 \\ 3 & -2 & 7 \end{pmatrix} \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 5 & 5 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{\frac{1}{5}R_2} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow[-R_2+R_3]{R_2+R_1} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \therefore \text{ LD vectors } \begin{pmatrix} \alpha + 3\gamma = 0 \\ \beta + \gamma = 0 \end{pmatrix}$$

They need not say that (this time!)
$-3\gamma(1,2,3) - \gamma(-1,3,-2) + \gamma(2,9,7) = (0,0,0)$
for all $\gamma \in \mathbb{R}$.