

Math 2311 Quiz #2

SPRING SEMESTER 2008

Name SOLUTIONS

1. Let $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 4 & -1 & 8 \end{pmatrix}$. Let $u = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$, $b = \begin{pmatrix} -8 \\ 2 \\ 22 \end{pmatrix}$, $c = \begin{pmatrix} -2 \\ -3 \\ -6 \end{pmatrix}$.

(a) Compute, and simplify, Au .

$$Au = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 4 & -1 & 8 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \\ 43 \end{pmatrix}$$

(b) Is there a solution to $Ax = b$? If so, find all solutions. If not, explain why not.

$$\begin{pmatrix} 1 & 2 & -1 & -8 \\ 2 & 1 & 2 & 2 \\ 4 & -1 & 8 & 22 \end{pmatrix} \xrightarrow[-4R_1+R_3]{-2R_1+R_2} \begin{pmatrix} 1 & 2 & -1 & -8 \\ 0 & -3 & 4 & 18 \\ 0 & -9 & 12 & 54 \end{pmatrix} \xrightarrow{3R_1} \begin{pmatrix} 3 & 6 & -3 & -24 \\ 0 & -3 & 4 & 18 \\ 0 & -9 & 12 & 54 \end{pmatrix}$$

$2R_2+R_1$
 $-3R_2+R_3$

$$\begin{pmatrix} 3 & 0 & 5 & 12 \\ 0 & -3 & 4 & 18 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{matrix} 3x+5z=12 \\ -3y+4z=18 \end{matrix} \quad \text{so every solution has the form}$$

$$(x, y, z) = \left(\frac{12-5z}{3}, \frac{4z-18}{3}, z \right)$$

ie/ $A \begin{pmatrix} \frac{12-5z}{3} \\ \frac{4z-18}{3} \\ z \end{pmatrix} = \begin{pmatrix} -8 \\ 2 \\ 22 \end{pmatrix}$

(c) Is there a solution to $Ax = c$? If so, find all solutions. If not, explain why not.

$$\begin{pmatrix} 1 & 2 & -1 & -2 \\ 2 & 1 & 2 & -3 \\ 4 & -1 & 8 & -6 \end{pmatrix} \xrightarrow[-4R_1+R_3]{-2R_1+R_2} \begin{pmatrix} 1 & 2 & -1 & -2 \\ 0 & -3 & 4 & 1 \\ 0 & -9 & 12 & 2 \end{pmatrix} \xrightarrow{-3R_2+R_3} \begin{pmatrix} 1 & 2 & -1 & -2 \\ 0 & -3 & 4 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

inconsistent

No solution to $Ax=c$.

2. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be defined by

$$T(x, y, z) = (3x - 2y + z, -x + 7y - 2z).$$

(a) Calculate $T(1, 2, 3)$.

$$T(1, 2, 3) = (2, 7)$$

(b) Find the standard matrix representation A of T .

$$A = \begin{pmatrix} 3 & -2 & 1 \\ -1 & 7 & -2 \end{pmatrix}$$