Math 2311 Quiz #1

SPRING SEMESTER 2008

Name SOLUTIONS

1. Let $A = \begin{pmatrix} 1 & 2 & -3 & -2 & 4 & 1 \\ 2 & 5 & -8 & -1 & 6 & 4 \\ 1 & 4 & -7 & 5 & 2 & 8 \end{pmatrix}$.

(a) Put A into *RREF* form.

$$A \xrightarrow[-R_1+R_3]{-2R_1+R_2} \begin{pmatrix} 1 & 2 & -3 & -2 & 4 & 1 \\ 0 & 1 & -2 & 3 & -2 & 2 \\ 0 & 2 & -4 & 7 & -2 & 7 \end{pmatrix} \xrightarrow[-2R_2+R_3]{-2R_2+R_1} \begin{pmatrix} 1 & 0 & 1 & -8 & 8 & -3 \\ 0 & 1 & -2 & 3 & -2 & 2 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{pmatrix}$$

$$\xrightarrow[8R_3+R_1]{-3R_3+R_2} \begin{pmatrix} 1 & 0 & 1 & 0 & 24 & 21 \\ 0 & 1 & -2 & 0 & -8 & -7 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{pmatrix}$$

(b) Solve the system of equations

$$\begin{aligned} x_1 + 2x_2 - 3x_3 - 2x_4 + 4x_5 &= 1 \\ 2x_1 + 5x_2 - 8x_3 - x_4 + 6x_5 &= 4 \\ x_1 + 4x_2 - 7x_3 + 5x_4 + 2x_5 &= 8 \end{aligned}$$

for all solutions (in parametric form).

x_1, x_2, x_4 : basic variables

x_3, x_5 : free variables

From RREF form, $x_1 = -x_3 - 24x_5 + 21$

$x_2 = 2x_3 + 8x_5 - 7$

$x_4 = -2x_5 + 3$

So every solution has the form

$$(x_1, x_2, x_3, x_4, x_5) = (-x_3 - 24x_5 + 21, 2x_3 + 8x_5 - 7, x_3, -2x_5 + 3, x_5)$$

$$= (21, -7, 0, 3, 0) + x_3(-1, 2, 1, 0, 0) + x_5(-24, 8, 0, -2, 1)$$