

Math 2311 Quiz #1

FALL SEMESTER 2007

Name SOLUTIONS

1. Let $A = \begin{pmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} -1 & -2 & -1 & 3 & 1 \\ 0 & -3 & -6 & 4 & 9 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{pmatrix}$

(a) Put A into RREF form.

$\xrightarrow{\substack{-2R_1+R_3 \\ R_1+R_4}} \begin{pmatrix} -1 & -2 & -1 & 3 & 1 \\ 0 & -3 & -6 & 4 & 9 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 2 & 4 & -6 & -6 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} -1 & -2 & -1 & 3 & 1 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & -3 & -6 & 4 & 9 \\ 0 & 2 & 4 & -6 & -6 \end{pmatrix}$

$\xrightarrow{\substack{2R_2+R_1 \\ 3R_2+R_3 \\ -2R_2+R_4 \\ \frac{1}{5}R_3}} \begin{pmatrix} -1 & 0 & 3 & -3 & -5 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\substack{3R_3+R_1 \\ 3R_3+R_2}} \begin{pmatrix} -1 & 0 & 3 & 0 & -5 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$\xrightarrow{-R_1} \begin{pmatrix} 1 & 0 & -3 & 0 & 5 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x_1 = 3x_3 + 5 \\ x_2 = -2x_3 - 3 \\ x_4 = 0 \end{cases}$

(b) Solve the system of equations

$$\begin{aligned} -3x_2 - 6x_3 + 4x_4 &= 9 \\ -x_1 - 2x_2 - x_3 + 3x_4 &= 1 \\ -2x_1 - 3x_2 + 3x_4 &= -1 \\ x_1 + 4x_2 + 5x_3 - 9x_4 &= -7 \end{aligned}$$

for all solutions (in parametric form).

Every solution has the form (x_1, x_2, x_3, x_4)

$$(3x_3 + 5, -2x_3 - 3, x_3, 0) = (5, -3, 0, 0) + x_3(3, -2, 1, 0)$$