

Math 2311 Quiz #11

SPRING SEMESTER 2008

Name SOLUTIONS

1. Let $A = \begin{pmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$.

(a) Find the least squares solution to $Ax = b$.

The normal equations associated with $Ax = b$ are

$$A^T A x = A^T b$$

ie $\begin{pmatrix} 1 & 1 & 1 \\ 3 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 3 & -1 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$

which simplify to $\begin{pmatrix} 3 & 3 \\ 3 & 11 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 14 \end{pmatrix}$

Since $\begin{pmatrix} 3 & 3 \\ 3 & 11 \end{pmatrix}^{-1} = \begin{pmatrix} 11/24 & -3/24 \\ 3/24 & 3/24 \end{pmatrix}$ we see that $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11/24 & -3/24 \\ -3/24 & 3/24 \end{pmatrix} \begin{pmatrix} 6 \\ 14 \end{pmatrix}$

ie the least squares solution is $\tilde{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(b) Find the orthogonal projection of $b = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$ onto the subspace

$$W = \text{span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \right\}.$$

$$\hat{b} = A \tilde{x} = \begin{pmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$$