Math 2311 Quiz #10

SPRING SEMESTER 2008

Name SOLUTIONS

1. Let $W = \{(-a+3b+c, a-b+c, 2a+2b+6c, a+b+3c) \mid a,b,c \in \mathbb{R}\}$.

(a) Explain - in clear precise terms - why W is a subspace of $\mathbb{R}^4$.

Note that $W = \{a(-1,1,2,1) + b(3,-1,2,1) + c(1,1,6,3) \mid a,b,c \in \mathbb{R}\}$
$= \text{span}\{(-1,1,2,1),(3,-1,2,1),(1,1,6,3)\}$

Since a span of vectors is a subspace, we see that W is a subspace.

(b) Find a basis for W. What is $\dim(W)$?

$$A = \begin{pmatrix} -1 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & 2 & 6 \\ 1 & 1 & 3 \end{pmatrix} \xrightarrow[\substack{2R_1+R_3 \\ R_1+R_4}]{R_1+R_2} \begin{pmatrix} -1 & 3 & 1 \\ 0 & 2 & 2 \\ 0 & 8 & 8 \\ 0 & 4 & 4 \end{pmatrix}$$

From this, we see that the original first 2 columns of A form a basis for W.

ie/ $\{(-1,1,2,1),(3,-1,2,1)\}$ is a basis for W.

& $\dim W = 2$.

(c) Is $(2,1,1,4) \in W$? If so, express $(2,1,1,4)$ as a linear combination of the basis elements that you found in part b. If not, explain why not.

$$\begin{pmatrix} -1 & 3 & 2 \\ 1 & -1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix} \xrightarrow[\substack{2R_1+R_3 \\ R_1+R_4}]{R_1+R_2} \begin{pmatrix} -1 & 3 & 2 \\ 0 & 2 & 3 \\ 0 & 8 & 5 \\ 0 & 5 & 6 \end{pmatrix} \xrightarrow[2R_4]{2R_1} \begin{pmatrix} -2 & 6 & 4 \\ 0 & 2 & 3 \\ 0 & 8 & 5 \\ 0 & 10 & 12 \end{pmatrix}$$

$$\xrightarrow[\substack{-4R_2+R_3 \\ -5R_2+R_4}]{-3R_2+R_1} \begin{pmatrix} -2 & 0 & -5 \\ 0 & 2 & 3 \\ 0 & 0 & -7 \\ 0 & 0 & -3 \end{pmatrix}$$

inconsistent so $(2,1,1,4)$ is not a LC of $(-1,1,2,1)$ & $(3,-1,2,1)$

ie/ $(2,1,1,4) \notin W$.

(d) Find a basis for $W^\perp$.

$(x,y,z,w) \in W^\perp \iff -x+y+2z+w=0$
$3x-y+2z+w=0$

$$\begin{pmatrix} -1 & 1 & 2 & 1 \\ 3 & -1 & 2 & 1 \end{pmatrix} \xrightarrow{3R_1+R_2} \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 2 & 8 & 4 \end{pmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{pmatrix} -1 & 1 & 2 & 1 \\ 0 & 1 & 4 & 2 \end{pmatrix}$$

$$\xrightarrow{-R_2+R_1} \begin{pmatrix} -1 & 0 & -2 & -1 \\ 0 & 1 & 4 & 2 \end{pmatrix}$$

ie/ $x = -2z - w$ so (x,y,z,w)
$y = -4z - 2w$ $= (-2z-w, -4z-2w, z, w)$
$= z(-2,-4,1,0) + w(-1,-2,0,$

$\therefore$ a basis for $W^\perp$ is $\{(-2,-4,1,0),(-1,-2,0,1)\}$.